Establishing of Directions in Plane and Error Value of the Horizontal Deformation Vector of the Constructions Corresponding to the Maximal Probability

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Abstract
Operation of the deformations and horizontal displacements measuring of the studied constructions, called for short horizontal deformations construction, the principal geodesic method is represented by the microtriangulation method. Based of the cyclical angular and linear of precision measurement, performed from the fixed points of the network, in the frame of this method, the horizontal vector in the control points on the studied construction is determined. In this work, a method of estabilshing of the directions in plane and value of the error of the vector of the horizontal deformation of the studied construction corresponding to the maximal probability is presented.

Keywords
Vector, deformation, point, microtriangulation, error, maximal probability.

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1. Introduction

The pursuit of the behaviour *in situ* of massive buildings refers to the changes of position and of shape on the whole or one some of their elements and informs the appearance of some evoluational phenomena which would affect the security of the building, called control points, to the fixed points situated on the stable grounds, out of the influence zone of building, forming the general system of reference.

The study of the buildings by geodetic methods is achieved, performing cyclic measurements (angular, linear, of level difference, etc.) from the fixed points outside of the building about the fixed points of the building. The frequency of the measurements is greater during the execution and into operation stage and it is more and more smaller as the deformations die away and the building is stabilized.

From the geodesic methods group, the microtriangulation is used in the survey of the behaviour *in situ* of the massive constructions and of the grounds round it as well. The measurements in each cycle are executed with the same precision as in the initial case. The compensation calculus is performed rigorously on the base of the mean square method, to obtain the most probable values of the horizontal deformation vectors in the all control points from the studied construction. Using the most adequate mathematical model, one can realize also a complete evaluation of the measurements precision and of deformation vectors.

The fundamentals of the functional model of the compensation by indirect measurements method, processed in matrix from are known [1], [2]. The mathematical model to evaluate the precision in plane of the horizontal deformation vectors, known at the present time was completed with new elements resulting of the last researches [3], … , [6].

2. The Algorithm and the Program of the Determination of the Horizontal Deformation Vector of the Studied Construction and Evaluation of the Precision

From the beginning it must be underlined that, as distinct from the case of determination of the position in plane of new points of the geodesic network, where the establishing of the optimal conditions of determination are obtained by analysing the minimum of the pedal curve generated by the ellipse of errors in every determined point, in the case of the determination of the horizontal deformation vector of a studied construction this is not possible. In this case, the error of the horizontal deformation vector depends, in the first place, on the position/direction of the vector with respect to the rectangular system of axes, in which the determinations are achieved. The system of rectangular axes *X*, *Y* is chosen so that it coincide with the principal axes of the studied construction or with the directions along which the solicitations are largest.

The microtriangulation network used in the survey of the behaviour of an arched dam is presented in Fig. 1. The structure of the network includes the following categories of points: control points or aiming marks (P_1 , P_2 ,…, P_N), fixed on the aval pavement of the barrage, station points (*I, II, III, IV*), from which the cyclical measurements are executed, points of reference (K_1 , K_2 , K_3) from which one determine possible changes in the position of the station points and points of orientation (O_1 , O_2 , O_3 , O_4) situated in the grounds with a high degree of stability are determined.

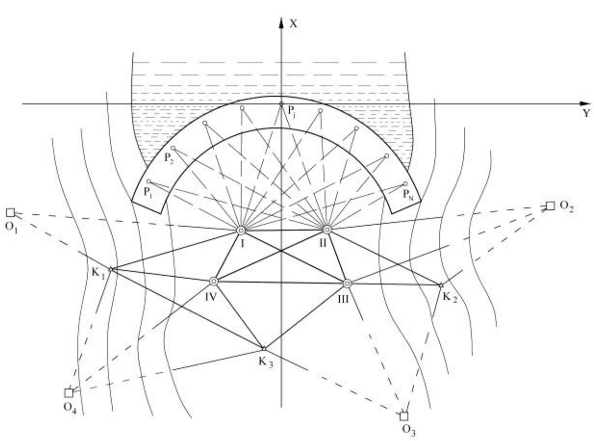

Fig. 1 Network of microtriangulation

Because of the difficult conditions in which one realizes the survey of the behaviour in time of the massive constructions, submitted to different solicitations from the fixed points of the microtriangulation network is realized, the most indicated method of treating of the measurements resulted from different cycles of observations is the rigorous method of the indirect measurements – Gh. Nistor algorithm [1]. In the frame of this algorithm one performs the determination of the horizontal deformation vector, simultaneously for a number of *N* control points, fixed on the construction, from a number of *P* fixed points of the network in function of *r* direct measurements on the ground, for example horizontal angles. This algorithm presents a character of wide generality being valid for different methods of determinations. By Gh. Nistor algorithm, are calculated, in the first place, the components of the horizontal deformation vector with the matrix relation [1]

$$X_{n1} = -Z_{nr}\, L_{r1}\ , \qquad (1)$$

or in developed form:

$$\begin{bmatrix} \Delta X_1 \\ \Delta Y_1 \\ \dots \\ \Delta X_N \\ \Delta Y_N \end{bmatrix} = -\begin{bmatrix} z_{11} & z_{12} & \dots & z_{1r} \\ z_{21} & z_{22} & \dots & z_{2r} \\ \dots & \dots & \dots & \dots \\ z_{n1} & z_{n2} & \dots & z_{nr} \end{bmatrix} \cdot \begin{bmatrix} l_1 \\ l_2 \\ \dots \\ l_r \end{bmatrix} . \qquad (2)$$

The constant matrix which appears in relations (1) and (2) is obtained from the elements of the initial cycle as a product

$$Z_{nr} = \tilde{Q}_{nn} B^T_{nr} P_{rr}\ , \qquad (3)$$

and the matrix of the free terms has as elements the differences between the horizontal angles measured in the initial/zero cycle and the actual one

$$l_i = \beta_{0i} - \beta_{1i} = \Delta\beta_i\ , \quad (i = \overline{1, r})\ . \qquad (4)$$

Based on the components ΔX_j , ΔY_j , $(j = \overline{1, N})$, the horizontal deformation vectors

$$L_{N,1} = L_{n/2,1} = \begin{bmatrix} L_1 \\ L_2 \\ \dots \\ L_N \end{bmatrix} = \begin{bmatrix} \sqrt{\Delta X_1^2 + \Delta Y_1^2} \\ \sqrt{\Delta X_2^2 + \Delta Y_2^2} \\ \dots\dots\dots\dots\dots\dots\dots \\ \sqrt{\Delta X_N^2 + \Delta Y_N^2} \end{bmatrix}, \qquad (5)$$

and its orientations in plane, corresponding to each control point,

$$\theta_{N,1} = \theta_{n/2,1} = \begin{bmatrix} \theta_{L_1} \\ \theta_{L_2} \\ \dots \\ \theta_{L_N} \end{bmatrix} = \begin{bmatrix} \arctan\,(\Delta Y_1 / \Delta X_1) \\ \arctan\,(\Delta Y_2 / \Delta X_2) \\ \dots\dots\dots\dots\dots\dots\dots \\ \arctan\,(\Delta Y_N / \Delta X_N) \end{bmatrix}, \qquad (6)$$

are calculated.

After the operation of compensation of the indirect measurements is performed, a correct and complete evaluation for the precision of the results is necessary.

The algorithm and program of evaluation of the precision in plane position of horizontal deformation vector will include following stages.

2.1. The Calculation of the Quadratic Mean Error of the Weight Unity

This represents the post-compensated error and it is calculated by generalized Bessel formula [2], [3]

$$s_0 = \pm\sqrt{\frac{V^T PV}{r-n}} \quad , \tag{7}$$

where V^TPV – the sum of square corrections which is calculated by Legendre-Gauss function:

$$V_{1r}^T P_{rr} V_{r1} = L_{1r}^T P_{rr} L_r + X_{1n}^T B_{nr}^T P_{rr} L_{r1} \ . \tag{8}$$

The value of the weight unity characterizes the conditions of measuring, that is the post compensation precision of the measurements.

2.2. The Calculation of Mean Square Errors along the Axes of Components and of Total Error

The precision evaluation of the components of the horizontal deformation vector in every control point of the construction is achieved using the variance–covariance matrix [2]

$$s_X^2 = s_0^2 \tilde{Q}_X \quad , \tag{9}$$

where $\tilde{Q}_X$ is the matrix of the cofactors (matrix of the weight coefficients of the unknowns/components) and has the form:

$$\tilde{Q}_X = \begin{bmatrix} Q_{X_1X_1} & Q_{X_1Y_1} & \dots & Q_{X_1X_N} & Q_{X_1Y_N} \\ Q_{X_1Y_1} & Q_{Y_1Y_1} & \dots & Q_{X_NY_1} & Q_{Y_1Y_N} \\ \dots & \dots & \dots & \dots & \dots \\ Q_{X_1X_N} & Q_{X_NY_1} & \dots & Q_{X_NX_N} & Q_{X_NY_N} \\ Q_{X_1Y_N} & Q_{Y_1Y_N} & \dots & Q_{X_NY_N} & Q_{Y_NY_N} \end{bmatrix} . \tag{10}$$

It represents the invers of the matrix of the coefficients of the normal ecuations ($\tilde{Q}_X = N_X^{-1}$) and it is a square matrix.

The square mean errors of the components of the horizontal deformation vector in the control point *j*, ($j = \overline{1, N}$), are expressed by

$$s_{\Delta X_j} = \pm s_0 \sqrt{Q_{X_jX_j}} \ , \ s_{\Delta Y_j} = \pm s_0 \sqrt{Q_{Y_jY_j}} \ , \tag{11}$$

and the mean error of the vector (total error) will be

$$s_{L_j} = \sqrt{s_{\Delta X_j}^2 + s_{\Delta Y_j}^2} \ . \tag{12}$$

Because it is found that the square mean errors do not characterize so well the precision, it is established [2] that the domain, in which the deformation vector from each control point will be situated, is represented by pedal curve, generated by the ellipse of errors.

2.3. The Ellipse of Errors and the Pedal Curve

It is established by study of the directions on a plane along which the mean square errors are maximum and minimum, of their values as well as of variation of the error along various directions round about horizontal deformation vector, that the domain in which every vector will be situated, with a given probability, is represented by the pedal curve generated by the ellipse of errors.

The determination of the orientations of the ellipse of errors for the control point *j*, is calculated using the trigonometric equation

$$\theta_j = \frac{1}{2}\arctan\frac{2Q_{X_jY_j}}{Q_{X_jX_j} - Q_{Y_jY_j}} \ . \tag{13}$$

The roots of this equation, denoted by θ_j and $\theta_j \pm 100^g$, determine the mutual orthogonal directions along which the error in horizontal deformation vector are maximum, and respectively, minimum. The two angles represent the orientations of the axes X^A, Y^A of the ellipse of error in comparison with the point has been determined (Fig. 2). The orientation angle of the great axis will appertain to [0^g, 200^g] interval.

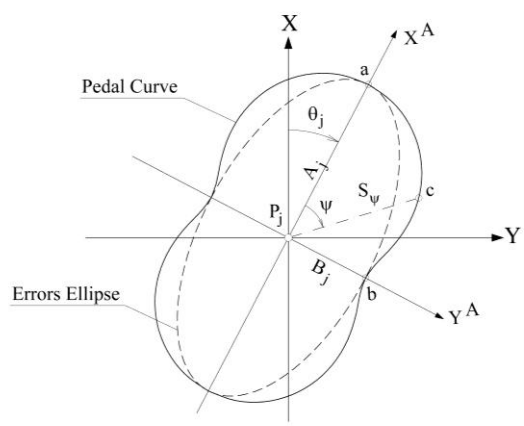

Fig. 2 Errors ellipse and pedal curve

The maximal and minimal value of the errors in the plane position of the points correspond to the semiaxes of the ellipse of errors. The values of maximal and minimal errors, given by the semiaxes, are calculated with the aid of relations:

$$A_j = \pm s_0\sqrt{\frac{1}{2}(Q_{X_jX_j} + Q_{Y_jY_j} + q_j)} \ ,$$
$$B_j = \pm s_0\sqrt{\frac{1}{2}(Q_{X_jX_j} + Q_{Y_jY_j} - q_j)} \ . \tag{14}$$

For the point *j* first is calculated the coefficient

$$q_j = \sqrt{(Q_{X_jX_j} + Q_{Y_jY_j})^2 + 4Q^2_{X_jY_j}} \;,\; q_j > 0 \;. \tag{15}$$

The drawing, at a superunit scale of the pedal curve generated by the ellipse of errors, with the help of equation

$$S^2_\psi = A_j^2 \cos^2\psi + B_j^2 \sin^2\psi \;, \tag{16}$$

where ψ represents the angle between axis X^A and any direction, considered in direct sense and contained in [0, 2π] interval. Equation (16) offers the expression of the vector radius of the pedal, representing the error of the vector along the considered direction

$$S_\psi = \sqrt{A_j^2 \cos^2\psi + B_j^2 \sin^2\psi} \;. \tag{17}$$

The pedal curve, generated by the ellipse of errors having the area expressed (Gh. Nistor, Gh. Andricioaei, 1994) by

$$A_{P_j} = \frac{\pi}{2}(A_j^2 + B_j^2) \;. \tag{18}$$

Analysing the relation (18), we conclude that for a given ellipse, the pedal curve area can takes different values as function of the semiaxes *A* and *B* ratio. For example, for the small semiaxis *B* area of the pedal curve is a function of the form

$$A_{P_j} = \frac{\pi}{2} B_j^2 \left[\left(\frac{A_j}{B_j} \right)^2 + 1 \right] \;. \tag{19}$$

The fact that for an ellipse of errors concerning the determination of the horizontal deformation vector, of given area, will result pedals of different areas, corresponding to the ratio of semiaxes, it means that also the probability of situation of the deformation vector on the surface of each pedal will be different, and certainly more than in the case of situation in the domain of ellipse of errors.

2.4. The Establishment of the Extremum Value Taken by the Probability, Corresponding to Different Directions Round About the Control Point

In [3] is established a relation which permits to compute the probability that the deformation vector belong to the domain represented by the pedal curve generated by the ellipse of errors (Gh. Nistor, Gh. Andricioaei, 1996)

$$P_{p_j} = 1 - \frac{1}{2\pi} \int_0^{2\pi} \exp\left[-\frac{1}{2} \cdot \frac{A_j^4 \cos^2\psi + B_j^4 \sin^2\psi}{(A_j^2 \cos^2\psi + B_j^2 \sin^2\psi)^2} \right] d\psi \;. \tag{20}$$

For the calculation of the probability one can use a computer program or the Simpson's approximation formula [6]. Relation (20) being difficult to access, one recommends to use the minimum and maximum values between which will be situated P_p , that is the probability that the horizontal deformation vector will be situated within the domain represented by the pedal curve of the errors ellipse. This will be expressed by inequalities [4], [5]:

$$1 - \exp\left(-\frac{1}{2}\right) \le P_{p_j} \le 1 - \exp\left[-\frac{(A_j^2 + B_j^2)}{8A_j^2 B_j^2} \right] . \tag{21}$$

The maximum value of the probability represented by the third term of (21) depends on the values of the semiaxes, more exactly on their ratio. To make obvious the influence of the ratio of semiaxes over the maximum of probability, the inequalities (21) can be written in the form [3]

$$0{,}3935 \le P_{p_j} \le 1 - \exp\left[-\frac{1}{8} \left(\frac{A_j}{B_j} + \frac{B_j}{A_j} \right)^2 \right] . \tag{22}$$

The minimum values of the probability for the horizontal deformation vector of the control point are obtained along the directions [5], (Fig. 3):

$$\psi_{\min} = 0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2},\ 2\pi \;, \tag{23}$$

and they correspond to the probability that vector be situated in the domain of the ellipse of errors ($P_p = P_e$ = 0,3935). In respect to *X* axis of the coordinate system, in which the control points are determined, the directions will be:

$$\theta_j;\ \theta_j + \frac{\pi}{2};\ \theta_j + \pi;\ \theta_j + \frac{3\pi}{2};\ \theta_j = 2\pi \;. \tag{24}$$

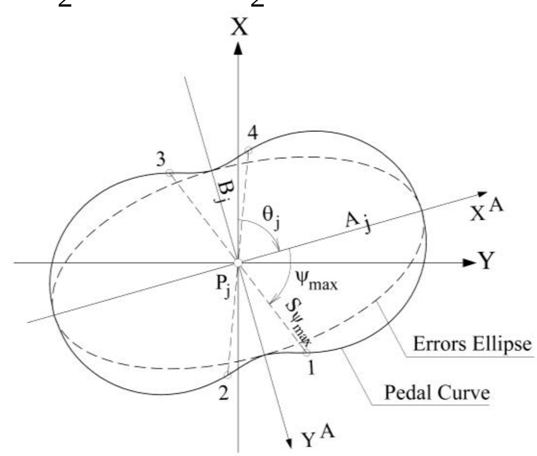

Fig. 3 Directions in plane and of the magnitude of the error of the horizontal deformation vector according to the maximal probability $S_{\psi_{\max}} \to \overline{P_j - 1};\ \overline{P_j - 2};\ \overline{P_j - 3};\ \overline{P_j - 4}$.

The other points of extremum, which correspond to the directions along which the probability under discussion is maximum

$$\psi_{\max};\ \pi - \psi_{\max};\ \pi + \psi_{\max};\ 2\pi - \psi_{\max} \;, \tag{25}$$

or, in respect to intial axis, *X*,

$$\theta_j + \psi_{\max};\ \theta_j + \pi - \psi_{\max};\ \theta_j + \pi + \psi_{\max};\ \theta_j + 2\pi - \psi_{\max} \;. \tag{26}$$

In the expressions (25) and (26), $\psi_{\max}$ is calculated with relation [5]:

$$\psi_{max} = \arccos\sqrt{\frac{B_j^2}{A_j^2 + B_j^2}} \quad , \tag{27}$$

where the angle ψ_{max}, between axis X^A and the direction under consideration, is computed with [4], [5].

2.5. The Calculation of the Mean Square Errors Corresponding to the Directions with the Maximum of the Probability so that the Horizontal Deformation Vector be Situated in a Plane

Along the four directions, defined by the angular values (25), the maximum of the probability under consideration is computed with the relation [4]

$$P_{p_{max}} = 1 - \exp\left[-\frac{1}{8}\left(\frac{A_j}{B_j} + \frac{B_j}{A_j}\right)^2\right] \quad , \tag{28}$$

which represents the third term of inequality (22). Along the direction with maximum probability one can compute the square mean error of the horizontal deformation vector of the studied construction with the expression of the vector radius of the pedal curve

$$S_{\psi\,max} = \sqrt{A_j^2 \cos^2\psi_{max} + B_j^2 \sin^2\psi_{max}} \quad . \tag{29}$$

The maximal probability assume values between 0,3935 and 1,000 and in these cases their square errors are very closely to values of small semiaxis B_j, which is remarkably. The variation probabilities of the control point, corresponding to variations lying on [0, 2π] interval. We can observe the points of extremun of the probability, the minimal and the maximal values corresponding to the magnitude of the axes ψ, for each ratio.

2.6. Application

Considering when the ratio of the semiaxes is A_j / B_j = 7.6mm / 2.5mm = 3.04, the probability that the horizontal deformation vector to lie in the pedal domain will be maximum and equal to P = 0.7579 (28), along the directions represented by angles (25): $\psi_1 = 79^g 76^c, \psi_2 = 120^g 24^c, \psi_3 = 279^g 76^c$ *and* $\psi_4 = 320^g 24^c$, where the angle ψ_{max} is calculated with the relation (27). Along the four directions, expressed by (28), the probability under discussion will be maximal, the square mean errors of the deformation vector will be equal to $S_{\psi_{max}} = 3.36mm$, very close to the minimum error represented by the small semiaxis, B_j = 2.5 mm. This result is remarkable only when the horizontal deformation vector of the studied construction, corresponds to the one of these directions.

3. Conclusions

The determination of the horizontal deformation vector from the control points from the studied construction, is obligatory to analyse the possibilities to obtain the smallest errors along the directions in plane of the deformation vectors. This is possible only in the case when are known with anticipation the directions in plane along which will be produced the horizontal deformations and displacements.

This wish will be realized in the frame of two operations:

a) Determination through projection of the configuration and orientation in plane of each pedal, so that the small semiaxis to coincide with the direction of the deformation vector by studying the disposition of the station points. In this manner will be influenced the values of the elements of the cofactor matrix (the weight square and rectangular coefficients) of the vector components on axis *X*, *Y* with respect to which the determinations are achieved.

b) The increment of the precision of the cyclical geodesic measurements which will lead to the increment of the precision of the cyclical angular and linear differences, and implicitly to the increment of the precision of the deformation vector.

At the level of whole construction one recommend to achieve a statistical study of the results, with a given probability, the state of efforts and deformations of the construction under study.

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