Experimental study regarding astro-geodetic vertical deviations

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Abstract
The main goal of the present study consist in the evaluation of the vertical deviations obtained by visual astro-geodetic technique. Were performed five series of observations, both azimuthal and zenithal, in different nights by different observers. All angular measurements were visual effectuated with a high precision and motorized electronic total station and time measurements with a manual electronic chronometer. All observations was adjusted separately, first time in-block (azimuthal and zenithal measurements together) and second time only zenithal measurements. Finally, resulted vertical deviations components were inter-compared and also compared with similar values obtained by different methods and instruments. The study served mainly to check the variation of vertical deviations components and associate precision parameters at some measurements errors or modification of input data.

Keywords
Astro-geodesy, vertical deviations, geoid.

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1. Introduction

Astro-geodetic vertical deviations are quantities of interest for geoid modeling, reduction of terrestrial measurements and other scientifically and practically applications.

Usually, the vertical deviation is decomposed in two orthogonal components: first, a meridian component (North-South direction), second a prime vertical component (East-West direction). The total vertical deviation represents the root square of the sum of squares of components.

The study is mainly focused on the investigation of the time errors effect (random or systematic, due to the operator or used instruments) in procedures of visual astro-geodetic observations.

2. Instruments and measurements

Observations were performed on a concrete pilaster located on the Bucharest Faculty of Geodesy roof, equipped with forced centering, that ensured a good stability needed in this case. We used a new and automatic Electronic Total Station (ETS), *Topcon MS05AX* gifted by bent eyepiece, illuminated reticular wires, LCD display and keyboard, useful facilities for night observations (Fig. 1). Also, used ETS has a 30X magnification, dual axes liquid compensator, 0".1 angular resolution and 0".5 angular accuracy. Time measurement was realized by a manual electronic chronometer (*Ruhla*) in the *UTC* (Universal Coordinated Time) time scale. The startup of the chronometer was visually realized with the help of a computer connected at an atomic time server (swisstime.ethz.ch) by a mobile internet connection and a sync software (*DS Clock* v.2.6.3 http://www.dualitysoft.com /dsclock). For all observed stars, at the end of observations was measured atmospheric pressure (mmHg) and temperature (^{o} Celsius) by an electronic meteo-station, for astronomical refraction calculus in the case of zenithal distances measurements.

All measurements (angular and time values, atmospheric parameters) were stored in a terrain laptop. Also, permanently, during the observations time, on the laptop run a planetarium type software (*Stellarium* v.0.12.4, www.stellarium.org) which gave us a real time sky map for easy choice of observed stars, facilitating uniform azimuthal distribution and speed of transition between the 2 positions of the ETS.

Fig. 1 Visual astro-geodetic observations effectuated with *Topcon MS05AX* on the concrete pilaster situated on the roof of the Faculty of Geodesy, Technical University of Civil Engineering Bucharest.

The first terrain operation was to orient the ETS in space. The leveling procedure was easy to realize by the dual-axes compensator indications or by the instrument levels, the vertical axis of the instrument being aligned with Zenith-Nadir direction (local vertical or plumb line). The horizontal orientation was made by bisecting *Polaris* with the vertical reticular wire, retaining the time of transit. For this time was rapidly calculated the *Polaris* azimuth (by tangent formula), value which was introduced at ETS horizontal circle. Thus, the zero horizontal circle was aligned on the South direction, obviously with a certain accuracy.

Every star was observed according to the following procedure: in the first instrument position (I), 3 azimuthal lectures together with corresponding times, 3 zenithal distance lectures together with corresponding times, next, in the second instrument position (II), similar with the first instrument position, another 6 angular values and 6 times values. Hence, for every star were taken 12 angular values, 12 corresponding times and 2 atmospheric parameters, usually at the final of observations.

Totally, there were effectuated 4 nights of observations, resulting 5 series of measurements. Measurements was performed by 3 different operators (denoted by G, M and A after the first letter of their name) using the same instruments and procedures. In the present study frame were observed in total 107 stars, effectuating a number of 2762 measurements (angular, times and atmospheric parameters). The effective time of observations was of $15^h.58$. The observations volume by every night is showed in Tables 1, 2 and 3.

Table 1 Number of nights, series and stars observed by every operator.

Nights	Date	Series	Operator	No. of stars
1	11.09.2014	1	A+M+G	14
2	12.09.2014	2	M/A	20/10
3	18.09.2014	3	A	21
4	19.09.2014	4	M	21
		5	A	21
TOTAL	4	5	3 (A, M, G)	107

Table 2 Number of observations by type (A. denote azimuthal measurements, z. denote zenithal measurements, Atm. represent atmospheric parameters).

Series	A. meas.	A. time meas.	z. meas.	z. time meas.	Atm.	Total meas.
1	84	84	84	84	28	364
2	180	180	177	177	60	774
3	123	123	123	123	42	534
4	126	126	126	126	42	546
5	126	126	125	125	42	544
TOTAL	639	639	635	635	214	2762

Table 3 Effective observation time interval for every series and the average time observation for a single star, both expressed in hours.

Series	No. of stars	Start (h)	Stop (h)	Obs. Time (h)	Average time/star (h)
1	14	17.54	20.41	2.87	0.21
2	30	17.87	21.93	4.07	0.14
3	21	16.78	20.67	3.89	0.19
4	21	16.66	19.09	2.43	0.12
5	21	19.85	22.17	2.32	0.11
TOTAL observation time/Average time on a single star				15.58	0.15 (~ 9^m)

Table 4 shows a sample of differences between azimuthal and zenithal measurements and corresponding calculated values, effectuated for 4 stars disposed in the vicinity of the cardinal points. For calculated values was used *Multiyear Interactive Computer Almanac* (*MICA* v.2.0, Fig. 2)

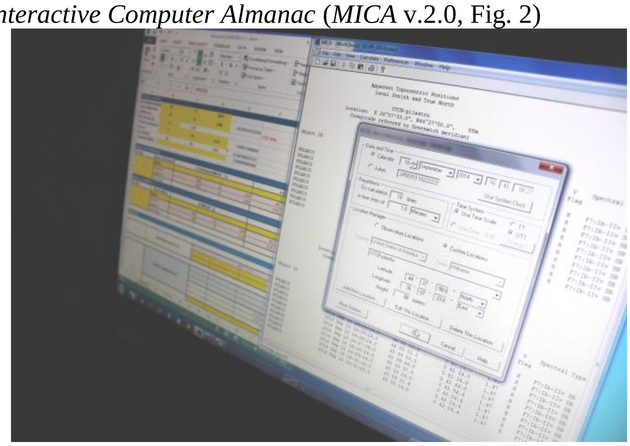

Fig. 2 Entire astrometry calculus effectuated with *MICA* software in terrain.

Table 4 Differences between measured azimuth and zenithal distances, and the corresponding calculated values by *MICA*.

Star	ETS position	Δ*A*"	Δ*z*"
Polaris Position: North, Operator: G Date: 11.09.2014	I	1.6	15.7
		7.0	8.3
	II	-3.0	6.6
		-1.5	8.2

		-2.8	10.4
		-8.3	6.1
Sheratan Position: East Operator: A Date: 12.09.2014	I	-24.7	-0.2
		-25.4	116.0
		-21.2	4.0
	II	-26.1	2.4
		-29.8	1.0
		-23.2	4.9
Altair Position: South Operator: M Date: 19.09.2014	I	-36.5	-7.9
		-34.6	-6.7
		-30.9	-7.0
	II	-34.8	-11.6
		-20.3	-8.5
		-25.8	-16.6
Vega Position: West Operator: M Date: 19.09.2014	I	-7.8	-1.7
		-11.2	-2.9
		-11.0	-4.5
	II	-5.3	-9.5
		-7.8	-6.9
		-5.9	-2.8

3. Data adjustment

All observations series was adjusted by 2 different methods. It was adjusted separately zenithal observations using indirect measurements functional-stochastic model, and next, it was adjusted in-block, both azimuthal and zenithal observations, using conditional with unknowns functional-stochastic model (general adjustment case).

For zenithal observations, starting from the cosines formula of the zenithal distance (Atudorei M., 1983):

$$F_z = \sin\Phi \cdot \sin\delta + \cos\Phi \cdot \cos\delta \cdot \cos H - \cos z\,, \quad (1)$$

results the linear form of correction equation (2), used only for zenithal observations adjustment:

$$\left(\frac{\partial F_z}{\partial \Phi}\right) d\Phi + \left(\frac{\partial F_z}{\partial H}\right) d\Lambda = z_{measured} - z_{calculated}\,. \quad (2)$$

For azimuthal observations, was used the cotangent formula of the azimuth in the positional triangle of the star (Atudorei M., 1983):

$$F_A = \sin H \cdot \cot A + \cos\Phi \cdot \tan\delta - \sin\Phi \cdot \cos H\,. \quad (3)$$

Using equations (1) and (3), for the in-block adjustment, results the linear forms of conditional equations (4) and (5).

Linear equation for azimuthal measurement:

$$\left(\frac{\partial F_A}{\partial t_A}\right) v_{t_A} + \left(\frac{\partial F_A}{\partial A}\right) v_A + \left(\frac{\partial F_A}{\partial \Phi}\right) d\Phi + \left(\frac{\partial F_A}{\partial H}\right) d\Lambda + \left(\frac{\partial F_A}{\partial U}\right) dU + w_A = 0\,. \quad (4)$$

Linear equation for zenithal measurement:

$$\left(\frac{\partial F_z}{\partial t_z}\right) v_{t_z} + \left(\frac{\partial F_z}{\partial z}\right) v_z + \left(\frac{\partial F_z}{\partial \Phi}\right) d\Phi + \left(\frac{\partial F_z}{\partial H}\right) d\Lambda + w_z = 0\,. \quad (5)$$

In relations (1)-(5) were made following notation: $d\Phi$ = correction to the provisory value of the latitude; $d\Lambda$ = correction to the provisory value of the longitude; dU = correction for azimuthal orientation of the zero horizontal circle; $z_{measured}$ = average of the measured zenithal distances (total 6 lectures, 3 in the first position, 3 in the second position), corrected by the influence of the non-linear trajectory of the star and astronomical refraction (*Roelofs* formula); $z_{calculated}$ = the zenithal distance calculated for the average of the times lectures (total 6 lectures, 3 in the first position, 3 in the second position); v_{t_A} = correction for the average time of azimuthal observations; v_A = correction for measured azimuth; w_A = the free term of the conditional equation corresponding to the azimuthal measurements; v_{t_z} = correction for the average time of azimuthal observations; v_z = correction for zenithal distance measured ($z_{measured}$); w_z = the free term of the conditional equation corresponding to the zenithal distances measurements.

In the case of zenithal observations adjustment, every star will provide one equation with 2 unknowns, corrections for latitude and longitude ($d\Phi$, $d\Lambda$).

In the case of in-block observations adjustment (azimuthal and zenithal together), every star will provide 2 equation with 3 unknowns, corrections for latitude, longitude and azimuthal orientation of the zero horizontal circle ($d\Phi$, $d\Lambda$, dU).

For zenithal observations adjustment was not used weights, but for in-block adjustment was used weights for both angular and time measurements. Weights was established taking into account the star's position on the sky and the star's trajectory in the optical field of the telescope.

As start values used in all adjustment, we use coordinates determinated by GPS technology, referred to GRS80 reference ellipsoid: geodetic latitude $\phi = 44^\circ 27' 50''.138$ and geodetic longitude $\lambda = 26^\circ 07' 32''.979$.

For every series, we perform a statistical check after adjusting, regarding great measurements errors removal. We use 2 different statistical tests (denoted by T1 and T2) both based on normalized correction and *Student* distribution (Fotescu N., 1978; Săvulescu C., 2002). A summary of the stars eliminated by applying both tests (T1 and T2) after a single run, are given in Table 5.

In the classical astro-geodesy, time errors of star's transit at reticular wires was named *personal observer equation*. It was considered that this error strictly depended of observer. Moreover, it was important that personal equation to be constant. Before and after the measurement campaign, the observer shall determine the longitude in a reference point, using the same instrument and method as in campaign. The difference between the measured and reference longitude represented the personal equation (Stamatin I., 1961).

In our study, before starting and after finishing the observations, all operators estimated their personal time errors, by a very simple procedure.

Table 5 Remaining stars' number after applying both statistical test T1 and T2, based on normalized correction and *Student* distribution.

Series	No. of observed stars	In-Block T1	In-Block T2	Zenithal T1	Zenithal T2	No. of remaining stars
		The star no. eliminated by T1 and/or T2 tests				
1	14	2	-	-	-	12*
2	30	23	23	23	23	29
3	21	15	15	15	15	20
4	21	10	10	10	10	20
5	21	9	9	9	9	20

*one star was found with erroneous measurements and was not introduced in the adjustments, both zenithal and in-block.

This procedure consisted in stopping the chronometer used at observations, at regular time interval, usually 5 or 10 seconds. It was realized many series, results being showed in the Table 6 (RMS represents the root mean square of the difference between measured time intervals and reference time intervals).

We started from the assumption that the error determined as we described above, are found almost identically in time measurements of each operator (at the star's transit at one of reticular wires). Moreover, in this way we estimate only the personal observer time error, while astronomical longitude determination implies a range of errors (residual instrumental errors, clock error, etc., inclusive personal observer time error)

Table 6. Estimated values of personal equation of every operator, before and after all series measurements.

		Operator G (s)	Operator M (s)	Operator A (s)
Before	Min.	-0.21	-0.16	-0.17
	Max.	0.37	0.15	0.31
	Average	0.19	0.08	0.18
	RMS	0.22	0.09	0.20
After	Min.	-0.29	-0.30	-0.19
	Max.	0.82	0.30	0.20
	Average	0.06	0.01	-0.04
	RMS	0.20	0.10	0.07

Also, a total budget of estimated errors involved in astro-geodetic observations can be found in Table 7. The sight error is a random one and appear on any direction depending on the human eye visual acuity and instrument telescope magnification. The operator time error can be considered a systematic one and has been already estimated in Table 6. The chronometer start error is a systematic one too, and have the same value as operator time error. The horizontal orientation error was considered as a sum of the sight error, operator time error and chronometer start error, as maximal value (we remind that this procedure involve *Polaris* observations). All errors values was calculated both in arc seconds and time seconds.

In our study, for the observation point we use as reference values for vertical deviations $\xi = 10''.60$ and $\eta = 4''.88$ (Bădescu O., 2014). These values was obtained in the past with similar methods but different instrument (Leica TC2002). These values come from a long series of observations and was selected as mean reference values, having a good agreement with vertical deviations derived from global geoid models (EGM2008, GOCE).

Table 7. Estimated values of errors involved in astro-geodetic observations effectuated with a geodetic instrument, both expressed in arc seconds and time seconds.

Errors (absolute value)		Zenithal measurements	Azimuthal measurements
Sight (random)	"	1.33	
	s	0.09	
Operator time	"	1.50-3.00	
	s	0.10-0.20	
Chronometer start	"	0.00-3.00	
	s	0.00-0.20	
Horizontal orientation	"	5.83-7.33	
	s	0.39-0.48	
Instrumental errors	"	–	
	s	–	
∑ without random errors [interval]	"	[1.50-6.00]	[7.33-13.33]
	s	[0.10-0.40]	[0.48-0.89]
∑ with random errors [interval]	"	[2.83-7.33]	[8.66-14.66]
	s	[0.19-0.49]	[0.58-0.98]

Table 8 presents differences between calculated longitudes of in-block adjustment for every observations series and astronomical reference longitude $\Lambda_{ref} = 26^{\circ}07'39''.787$. Also, the astronomical reference latitude has the value $\Phi_{ref} = 44^{\circ}28'00''.738$.

Table 8 Differences between longitude obtained in every series observations, and reference longitude.

Series	$\Lambda - \Lambda_{ref}$ (")	$\Lambda - \Lambda_{ref}$ (s)	Operator
1	4.84	0.32	G+M+A
2	6.64	0.44	M+A
3	6.51	0.43	A
4	5.42	0.36	M
5	6.28	0.42	A
Average	5.94	0.39	-
RMS	5.98	0.40	-

It can be observed that differences between astronomical longitude obtained in every series observations and reference longitude (Table 8) are in accordance with the estimated values of errors involved in astro-geodetic observations effectuated with a geodetic instrument (Table 7). Moreover it can be observed in Table 8 that for every observations series, differences have the same sign (positive value). As we know, all time errors are found in the longitude in its entirety, so in Table 9 was calculated vertical deviations for which, all time measurements was corrected with corresponding values for every series (Table 8).

Although $s_{d\Phi}$ and $s_{d\Lambda}$ represent standard deviations resulted from adjustments and are referred to the astronomical coordinates Φ and Λ, however can be assigned almost in integrality to vertical deviations components ($s_{d\Phi} \approx s_{\xi}$ and $s_{d\Lambda} \approx s_{\eta}$). Table 9 shows results obtained after applying both statistical tests for large measurements errors removal (T1 and T2) and (systematic) time correction determined in Table 8. Table 10 shows differences between results with and without time corrections only, except T1 and T2 statistical tests for big measurements errors removal. Also, s_0 represents the adjustment standard deviation and s_{dU} the standard deviation of azimuthal orientation of the zero horizontal circle.

Table 9 Results obtained before (denoted by superscript *) and after (without superscript) applying both statistical tests for big measurements errors removal (T1 and T2) and (systematic) time corrections as determined in Table 8.

Series	Type of adjustment	ξ''	η''	s_0''	$s_{d\Phi}''$	$s_{d\Lambda}''$	s_{dU}''
1	In-block*	11.916	8.335	2.002	0.846	1.237	1.436
	In-block	11.214	4.446	1.518	0.668	1.019	1.095
	Zenithal*	12.243	7.301	3.175	1.127	1.998	
	Zenithal	12.236	3.869	3.169	1.125	1.994	
2	In-block*	11.278	10.379	2.220	0.709	0.962	0.924
	In-block	11.342	4.905	1.715	0.553	0.762	0.725
	Zenithal*	10.491	10.235	3.842	1.019	1.361	
	Zenithal	10.567	4.523	2.534	0.672	0.926	
3	In-block*	10.776	9.692	1.767	0.634	0.979	0.935
	In-block	11.283	4.907	1.516	0.560	0.857	0.832
	Zenithal*	9.976	9.066	2.831	0.812	1.341	
	Zenithal	10.586	4.132	2.194	0.652	1.046	
4	In-block*	10.228	9.800	2.000	0.754	0.927	0.977
	In-block	10.278	4.897	1.666	0.621	0.866	0.893
	Zenithal*	8.025	7.348	6.028	1.785	2.736	
	Zenithal	10.301	3.613	1.983	0.615	0.900	
5	In-block*	12.353	8.426	3.510	1.248	1.801	1.916
	In-block	12.127 (11.772)	5.516 (4.913)	1.699 (1.712)	0.621 (0.679)	0.898 (1.004)	0.935 (0.980)
	Zenithal*	11.163	9.288	3.313	0.988	1.487	
	Zenithal	11.932 (11.546)	5.728 (5.296)	1.963 (1.856)	0.599 (0.606)	0.905 (0.907)	

Values in parenthesis for the series 5 in the case of in-block was obtain by applying both statistical tests for big measurements errors removal (T1 and T2), (systematic) time corrections as determined in Table 8 and in addition to other series, by eliminating stars with the number 8,10 and 16. Stars 8 and 10 was eliminated because they had great free terms on zenithal measurements, comparatively with all other stars. As a consequence, stars 8 and 10 was eliminated of both adjustments. The star 16 was eliminated because it had a great free term on azimuthal measurements. As a consequence, star 16 was eliminated only from in-block adjustment.

Table 10 Differences between results with and without time correction.

Series	Type of adjustment	$\Delta\xi''$	$\Delta\eta''$	$\Delta s_0''$	$\Delta s_{d\Phi}''$	$\Delta s_{d\Lambda}''$	$\Delta s_{dU}''$
1	In-block	0.000	-3.434	0.001	0.003	0.000	0.000
	Zenithal	-0.007	-3.432	-0.006	-0.002	-0.004	0.000
2	In-block	-0.011	-4.711	0.000	0.000	0.000	0.000
	Zenithal	-0.015	-4.712	-0.001	-0.001	-0.001	0.000
3	In-block	-0.010	-4.617	0.001	0.000	0.000	0.000
	Zenithal	-0.013	-4.617	0.000	0.001	0.000	0.000
4	In-block	-0.005	-3.854	-0.001	-0.001	0.001	0.001
	Zenithal	-0.007	-3.854	-0.002	0.000	-0.001	0.000
5	In-block	-0.002	-4.450	-0.001	0.001	0.001	0.001
	Zenithal	-0.006	-4.454	0.000	0.000	0.000	0.000

Table 11 shows differences between the reference values for both components ξ and η and finals values obtained in every observations series.

Table 11 Differences between reference values and final values obtained in every series, for ξ and η

In-block adjustment						
Series	ξ''_{ref}	η''_{ref}	ξ''_i	η''_i	$\Delta\xi''$ (reference-series)	$\Delta\eta''$ (reference-series)
1	10.60	4.88	11.214	4.446	-0.614	0.434
2			11.342	4.905	-0.742	-0.025
3			11.283	4.907	-0.683	-0.027
4			10.278	4.897	0.322	-0.017
5			11.772	4.913	-1.172	-0.033
RMS					0.758	0.195
Zenithal adjustment						
Series	ξ''_{ref}	η''_{ref}	ξ''_i	η''_i	$\Delta\xi''$ (reference-series)	$\Delta\eta''$ (reference-series)
1	10.60	4.88	12.236	3.869	-1.636	1.011
2			10.567	4.523	0.033	0.357
3			10.586	4.132	0.014	0.748
4			10.301	3.613	0.299	1.267
5			11.546	5.296	-0.946	-0.416
RMS					0.856	0.835

4. Statistical analysis of results

For the quality evaluation of the vertical deviations determinations, it was applied a total of 7 statistical tests, regarding dispersions comparison, anomalous values elimination, comparisons between the 2 types of adjustments (measurements) and verifying the existence of systematic errors, both for in-block and zenithal only adjustments. Statistical evaluation was applied on vertical deviations components ξ and η as final results of our study, instead of astronomical coordinates Φ and Λ.

a. Dispersions comparison – *Bartlett* test (Ceaușescu D, 1973; Săvulescu C., 2002, Table 12).

In the case of in-block adjustment, for probabilities $P=95\%$ and $P=99\%$, the test indicate that between 5 standard deviations there is not a significant difference, both for ξ and η. This means that it can be used the simple arithmetic mean (6) and the (modified) standard deviation (7):

$$x_m = \frac{x_1 + x_2 + ... + x_n}{n}, \quad (6)$$

$$s_x = \sqrt{\frac{\sum_{i=1}^{i=n}(x_i - x_m)^2}{n-1}}, \quad (7)$$

where n is the number of determinations (ξ or η) and x_n represent values of ξ or η obtained in every series.

In the case of zenithal adjustment, for $P=95\%$ and $P=99\%$, the test indicate that between 5 standard deviation there is no significant difference for ξ and there is a significant difference for the η component. Because one component did not pass the *Bartlett* test we decide to apply the same solution for both component. This means, that we will use weight average (8) and standard deviation as weight average of the individual standard deviation depending on the number of measurements (9).

$$x_m = \frac{p_1 x_1 + p_2 x_2 + ... + p_i x_i}{p_1 + p_2 + ... + p_i};\ i = \overline{1,n}$$
$$p_i = 1 / s_{xi}^2 \quad (8)$$

$$s_x = \sqrt{\frac{(n_{x1}-1)\cdot s_{x1}^2 + ... + (n_{xi}-1)\cdot s_{xi}^2}{(n_{x1}-1) + ... + (n_{xi}-1)}} \quad (9)$$

where n is the number of determinations (ξ or η), s_{xi} is the standard deviation of each determination, x_m is the determinations average and n_{xi} represent the number of measurements for each determinations. Once again, all above parameters are refered to vertical deviations components ξ and η.

Table 12 The results of the *Bartlett* test, regarding mean value and standard deviation for the 2 types of adjustments.

	In-block (1)		Zenithal (2)		Differences (1) - (2) in absolut values	
	ξ''	η''	ξ''	η''	$\Delta\xi''$	$\Delta\eta''$
Mean value	11.178	4.814	10.877	4.375	0.301	0.439
Standard deviation	0.548	0.206	0.715	1.120	0.167	0.914

b. Elimination of anomalous (extreme) values from determinations (Ceaușescu D, 1973; Săvulescu C., 2002).

For this action we use three different statistical tests, as follows: *Grubbs*, *Q* and the *test based on the confidence interval*, with the specifications that the *Q* test is not even indicate for more than 4 determinations and the *test based on the confidence interval* is not considered very rigorously.

For in-block adjustment, *Grubbs* test indicate that for probabilities of $P=95\%$ and $P=99\%$, values $\xi_{\min}$, $\xi_{\max}$, $\eta_{\min}$ and $\eta_{\max}$ could not be considered anomalous values.

In the case of zenithal adjustment, *Grubbs* test indicate that for $P=95\%$, $\xi_{\max}$ is an abnormal value ($\xi_{\max} = 12''.236$), but for $P=99\%$ is a normal value. For both probabilities $P=95\%$ and $P=99\%$, $\xi_{\min}$, $\eta_{\min}$ and $\eta_{\max}$ are normal values.

For in-block adjustment, Q test indicate that for $P=95\%$ and $P=99\%$, $\xi_{\min}$, $\xi_{\max}$ and $\eta_{\max}$ are considered normal values, but $\eta_{\min}$ is an anomalous value ($\eta_{\min}=4''.446$). In the case of zenithal adjustment, for both probabilities $P=95\%$ and $P=99\%$, $\xi_{\min}$, $\xi_{\max}$, $\eta_{\min}$ and $\eta_{\max}$ are considered normal values.

The *test based on the confidence interval* showed that in the case of in-block adjustment, for $P=95\%$ and $P=99\%$, extreme values for ξ are out of the confidence interval, which means that these values can be considered anomalous ($11''.772$, $10''.278$). For η, both of $P=95\%$ and $P=99\%$, the minimum extreme is out of the confidence interval ($4''.446$).

In the case of zenithal adjustment, the test indicates that for $P=95\%$ minimum extreme values for ξ are out of the confidence interval ($10''.301$) and for $P=99\%$, the two largest values for ξ are out of the confidence interval ($11''.546$, $12''.236$). For η component, for $P=95\%$ the test indicates that extreme values are out of the confidence interval ($3''.613$, $5''.296$) and for $P=99\%$, no value can be considered anomalous.

As we can see, the *Grubbs* test indicate that no value, both for ξ and η should not be excluded as external value. Although Q and *Test based on confidence interval* indicates that there are some values that can be considered external, however those values do not pass tests at limit.

c. Comparing standard deviations for the two types of adjustments – *Fisher* test.

For $P=95\%$ and $P=99\%$, for values of ξ, the test indicates that there is no significant difference between the two adjustments types. For $P=95\%$ and $P=99\%$, for η component, the test indicate that there is a difference between the two adjustments (in block versus zenithal only).

This statistical test reveals that astronomical latitude and consequently ξ is less sensitive to number of observed stars, time errors and uniformity of azimuthal distribution.

We noticed that the difference between precisions of the 2 methods cannot be evidenced in the case of a small number of determinations, only if the difference is large that is the present case. If the difference is small, it can be evidenced only if we perform a greater number of determinations.

d. Comparing averages for the two adjustments – *Student* test (Ceaușescu D, 1973; Săvulescu C., 2002). For $P=95\%$ and $P=99\%$, for values of ξ and η, the test indicates that there is no significant difference between averages. The difference between values obtained by in-block and zenithal adjustments, both for ξ and η are a result of random errors. The result is important taking into account that the *Fisher* test indicated a difference between precisions of those 2 types of adjustments (η component).

e. Verify the existence of systematic factors acting on the results – *Method of successive differences* (it is important to note that in this test we use modified standard deviation, indifferent of the *Bartlett* test results) (Ceaușescu D, 1973; Săvulescu C., 2002). In the case of in-block adjustment, for $P=95\%$ and $P=99\%$, the test indicate that there is no source of significant systematic errors, both for ξ and η. In the case of zenithal adjustment, for $P=95\%$ and $P=99\%$ the test indicate that there is a source of significant systematic errors for ξ but not for η.

Although, the test indicates that there are significant errors in ξ component for zenithal adjustment, however do not pass tests at limit.

5. Remarks and conclusions

All measurements was effectuated by different operators with the same instruments, specifying that angular measurements was effectuated in good conditions of stability.

For all measurements, the instrument worked with dual-axis compensator and the automate correction of the instrumental errors activated. At the beginning of the measurement campaign, the instrument was verified according to manufacturer indications.

In all observations series was used the same procedure for horizontal orientation (by *Polaris* observations).

All angular measurements effectuated in both positions of the instrument was reduced to the "mean wire", both for azimuthal and zenithal observations by specific formulas. All zenithal measurements was corrected by astronomical refraction using *Roelofs* formula, which depend only of atmospheric pressure and temperature.

All time measured values in the *UTC* time scale was corrected by *DUT1* (=*UT1-UTC*) from *IERS* bulletins (http://toshi.nofs.navy.mil/).

For all measured stars we create sheets of calculus for observations reduction from catalogue epoch (J2000.0) to the time of observations (corrections of polar motion, precession, nutation, annual parallax and aberration as well as diurnal parallax and aberration). All those calculus was verified by *MICA*. Also, for the stars coordinates we use the *FK5* catalogue.

The statistical tests was applied only once despite the fact that there are no rules regarding the number of runs. We remarked that applying statistical tests until there are no stars to be eliminated, the precision increase (reaching relative unrealistic values) while the vertical deviations components typically can suffer relative small changes (Table 13). Also, running test more than one time leads to an un-uniform azimuthal distribution.

Table 13 ξ and η values evolution (case of zenithal measurements only) until statistical tests T1 and T2 indicate that no star have to be eliminated. Retained values in our study are found at first run (line 2 in the below table.

No. of runs	ξ''	η''	s''_0	$s''_{d\Phi}$	$s''_{d\Lambda}$
0	10.475	5.523	3.840	1.019	1.361
1	10.567	4.523	2.534	0.672	0.926

2	10.515	3.892	1.896	0.503	0.718
3	10.242	3.697	1.736	0.474	0.667
4	9.955	3.763	1.568	0.442	0.604
5	10.055	4.035	1.425	0.404	0.570
6	10.065	3.787	1.309	0.371	0.545

We tried to estimate systematic time errors, no by astronomical longitude determination (personal equation observer), but by simple time chronometer measurements. Results are close to those obtained from longitude difference (Table 8) taking into account that we use a reference value for longitude with a standard deviation s_0 of about $0''.8$. Applying (systematic) time corrections, practically do not modify ξ component (time errors have a small influence on the latitude) but change with the same quantity (transformed in arc seconds) the η component.

Table 14 bring together results obtained in our study and previous results obtained similar as procedure but with different instruments and operators (Bădescu O., 2014). For instance, in the case of Leica TCRP1201+ was not apply T1 and T2 statistical tests for large measurements errors removal, but several stars was eliminated depending on the free terms size and azimuthal distribution uniformity. Also, Table 14 shows a comparison between astro-geodetic determinations and value derived from global geoid models GOCE and EGM2008. Finally in this study we have not sought to obtain the "best results" but we try to obtain the "real results". For example, for this reason, we do not eliminated any value after applying statistical analysis (because most of them failed tests at limit), even if the elimination of some values would be much improved final results.

Table 14 Comparisons between results obtained with 3 different instruments in different years. On series, upper values are referred to zenithal measurements while lower values are referred to in-block measurements/adjustments (azimuthal + zenithal). For Topcon MS05AX are shown all value, indifferent of the results provided by statistical tests. Also, last 2 rows shown differences between vertical deviations components extracted from 2 global geoid models (GOCE: $\xi = 9''.44$, $\eta = 4''.78$; EGM2008: $\xi = 11''.24$, $\eta = 4''.04$) and similar results obtained from astro-geodetic determinations.

Series	① Leica TC2002 (2002)				② Leica TCRP1201+ (2012)				③ Topcon MS05AX (2014)			
	ξ''	η''	s_ξ''	s_η''	ξ''	η''	s_ξ''	s_η''	ξ''	η''	s_ξ''	s_η''
1	-	-	-	-	11.25	5.13	0.24	0.21	12.236	3.869	1.125	1.994
	10.09	5.01	1.01	1.01	-	-	-	-	11.912	4.904	0.844	1.271
2	-	-	-	-	10.16	5.27	0.30	0.42	10.567	4.523	0.672	0.926
	11.04	4.88	0.69	0.69	-	-	-	-	11.342	5.905	0.553	0.762
3	-		-	-	10.69	5.10	0.22	0.24	10.586	4.132	0.652	1.024
	10.19	5.64	0.93	0.94	-	-	-	-	11.282	4.907	0.516	0.560
4	-	-	-	-	10.48	6.31	1.22	2.28	10.301	3.613	1.615	0.900
	10.33	4.96	0.95	0.87	-	-	-	-	10.278	4.897	0.621	0.866
5	-	-	-	-	10.60	5.35	0.19	0.24	11.546	5.296	0.606	0.907
	10.49	4.86	0.77	0.80	-	-	-	-	11.772	4.913	0.679	1.004
6	-	-	-	-	-	-	-	-	-	-	-	-
	10.66	4.77	0.30	0.33	-	-	-	-	-	-	-	-
Max	-	-	-	-	11.25	6.31	1.22	2.28	12.236	5.728	1.615	1.994
	11.04	5.64	1.01	1.01	-	-	-	-	12.127	5.905	0.844	1.271
Min	-	-	-	-	10.16	5.10	0.19	0.21	10.301	3.613	0.606	0.900
	10.09	4.77	0.30	0.33	-	-	-	-	10.278	4.897	0.516	0.560
Max-Min	-	-	-	-	1.09	1.21	1.03	2.07	1.935	2.115	1.009	1.094
	0.95	0.87	0.71	0.68	-	-	-	-	1.849	1.008	0.328	0.711
Average*	-	-	-	-	10.64	5.43	0.43	0.68	11.124	4.373	0.934	1.150
	10.47	5.02	0.77	0.77	-	-	-	-	11.388	5.226	0.631	0.871
Final value**	-	-	-	-	10.71	5.20	0.42	0.72	10.877	4.375	0.715	1.120
	10.60	4.88	0.80	0.80	-	-	-	-	11.178	4.814	0.548	0.206
GOCE-①/②/③	-	-	-	-	-1.27	-0.42	-	-	-1.44	0.41	-	-
	-1.16	-0.10	-	-	-	-	-	-	-1.74	-0.03	-	-
EGM2008-①/②/③	-	-	-	-	0.53	-1.16	-	-	-0.15	-0.33	-	-
	0.64	-0.84	-	-	-	-	-	-	0.06	-0.77	-	-

*arithmetic mean; ** values resulted after applying *Bartlett* test

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