A comparative study between gravimetric method and geometric method about determination of a precise quasigeoid for the area of Romania

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Abstract
With the official launch of Romanian Position Determination System - ROMPOS, whose basic infrastructure consists of a network of 74 active geodetic reference station, the main problem of surveyors in Romania is to calculate a precise quasigeoid model.
The topic was addressed in several large studies, particularly doctoral thesis, noting that the absence of gravity measurements, gravimetric method was treated for only a limited area, namely the Bucharest zone.
Therefore, this article presents a comparative study between the geometric and gravimetric determination methods for a quasigeoid model, test area being limited to Bucharest.

Keywords:
GNSS, precise levelling, quasigeoid, gravimetry, physical geodesy.

1. Introduction

Achieving high precision geoid model is possible only through a unified and consistent policy at national level in this regard, by cooperation between specialized institutions in the country, such as profile universities, National Agency for Cadaster and Land Registry - ANCPI through its subordinate units, Romanian Geological Institute, Military Topographic Directorate and so on. Also, both interdisciplinary nature and high degree of complexity of such a project involves allocating substantial budgets to achieve the expected objectives.

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For the reasons stated above, the issue of determining a national geoid model or quasigeoid has long been neglected. However, over the years, some of surveying specialists in our country have made considerable efforts to determine some geoid models viable corresponding to national territory. These investigations were made in some isolated projects, in most cases being the subject of some doctoral dissertations. We list below some of the most important studies in the field, made in recent decades in Romania:

- Mihăilescu, M. – Doctoral thesis – ”Quasigeoid for Romania, determined through astronomic and geodetic methods” – 1974;
- Ioane, D. – Study of Geological Institute of Romania in partnership with Canadian specialists – ”Geoid for Romania based on global geopotential model OSU91”;
- Serediuc, C. – Doctoral thesis – ”Geoid for a test area which covers approximately 20% of Romanian territory, determined using the finite element method” – 1996;
- Marinescu, M., Tomoioagă, T. – Theme of scientific research developed in the Research Agency for Military Equipment and Technologies, beneficiary being the Military Topographic Directorate – ”Geoid for Romania based on global geopotential model EGM96 and military gravimetric network” – 1998;
- Rus, T. – Doctoral thesis – Studies and research on topo-geodetic precision positioning using satellite methods - Modern methods of determining the geoid” 2000;
- Spiroiu, I. – Doctoral thesis – ” Contributions to the development of methods for determining a high resolution geoid undulations for theree dimensional geodetic networks” – 2005;
- Tomoioagă, T. – Doctoral thesis – ”Contributions to determine geoid undulations using global geopotential models and local gravimetric” – 2007;
- Tomoioagă, T. – ”Considerations about precision of determination of gravimetric geoid” – 2008;
- Dumitru, P. D. – Doctoral thesis – ”Contributions for quasigeoid determination in Romania” – 2011;

- Sorta, V. – Doctoral thesis - ”Contributions for a precise geoid / quasigeoid model for the area of Romania” – 2013.

2. Determination of the quasigeoid model for the Bucharest area by gravimetric method

Quasigeoid model for Bucharest area is integrated in TransDatRO software and takes the form of a grid of height anomalies, with density of 15”x15”, grid limits being defined by rectangle of coordinates:

(λminim : 25.95671898°; φminim : 44.32533744°),

(λmaxim : 26.24838588°; φmaxim : 44.60033766°).

Data sources that were the basis for achieving quasigeoid model were:

- values of absolute gravity for 129 points, uniformly distributed on Bucharest and Ilfov County. 2 points were reference points in which were transmitted absolute gravity values by linking to the 1st order National Network from Surlari Observatory and Dealul Piscului Observatory;
- GNSS measurements in the 129 gravimetric points;
- normal heights in gravimetric points determined through 1st order geometric levelling;
- Bouguer anomalies for Romania, given by International Gravimetric Bureau (BGI);
- high resolution digital terrain model for Romania, in DTED-Level 2 format;
- digital terrain model for Europe from SRTM 4 mission;
- Global Geopotential Model EGM2008.

Gravimetric measurements were performed by specialists belonging to SC PROSPECȚIUNI S.A. and National Center for Cartography (CNC) using a Lacoste-Romberg G model gravimeter, gravimetric points are arranged as in Fig. 1. GNSS measurements were performed at the same points that were determined relative values of gravity, being carried out by teams of specialists from the CNC. These measurements were carried out with Trimble, model 4000 SSI receivers, observations realized for one hour sessions and with a registration rate for 10 seconds.

Precise geometric levelling measurements were performed in 2010, 2011 and 2012by specialists from CNC. For this purpose were used three tipes of levels: Carl Zeiss Jena Ni002 and Ni007 and Leica Sprinter 250M.

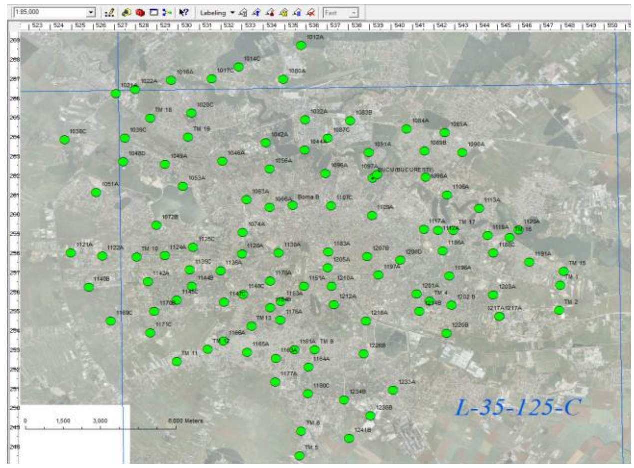

Fig. 1: The arrangement of gravimeric points (Source: CNC)

Below it can be seen the levelling network scheme:

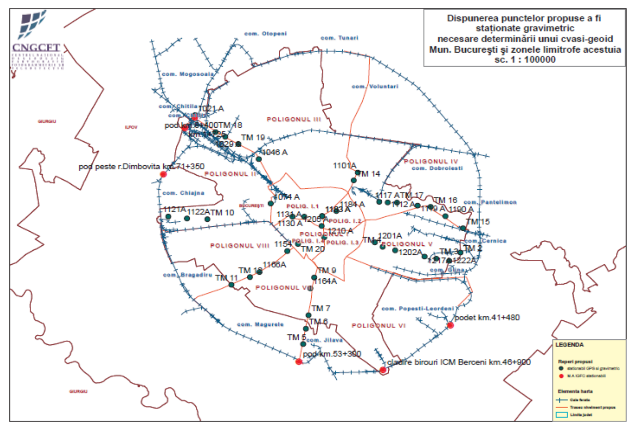

Fig. 2: The scheme of levelling network (Source: CNC)

Gravimetric data processing stages are divided into three areas of processing:

- precise gravity domain (in gravimetric points);
- gravity domain in the grid nodes, grid based on improved Bouguer anomalies for Romania, published by BGI;
- domain of height anomalies which results from applying the remove – restore algorithm.

The main stages of data processing to generate quasigeoid model were:

a. Preparation of digital terrain models to calculate terrain corrections and indirect effect.

Were used two digital terrain models:

- a more detailed model with a higher resolution in DTED Level 2 format that is used for close point computing;
- a less detailed model with lower resolution in DTED format from STRM4 (Shuttle Radar Topography Mission 4), which is used for the remote area of computed point.

b. Correction of improved Bouguer anomalies, published by BGI

Were performed the following steps:
- Coordinate transformation from Krasowski 1940 ellipsoid to GRS80 ellipsoid;
- Computing of normal gravity using Silva – Casinis formula;
- Computing of normal gravity using parameters and derived constants of GRS80 ellipsoid given by Moritz (1980) and the close formula of Somigliana;
- Computing of the new improved Bouguer anomalies in the grid nodes.

c. Calculation of Faye anomalies, terrain corrections and improved Bouguer anomalies in gravimetric points

For calculating terrain effect on the gravity anomalies, was used the homogenous rectangular prisms method combined by spherical harmonic formulas (Mc Millan). This depends on terrain geometry, grid density and distance from computed point. Maximum radius is 100km.

Terrain correction was computed using the two digital terrain models, taking into account the following general relation (Physical Geodesy, Nico Sneeuw, Stuttgart, 2006):

$$TC = G\int_x\int_y\int_{z=H_P}^{H}\frac{z-H_P}{r^3}\rho(x,y,z)\,dx\,dy\,dz \qquad (1)$$

where r is the distance to computed point P.
The implementation of this integral to a certain resolution digital terrain model can be like in the following figure:

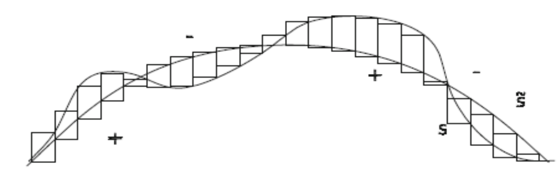

Fig. 3: The implementation of prisms method on finite elements of terrain

The evaluation of terrain correction for each finite element from the digital terrain model assume customizing of the relation no. (1) for a constant density rectangular prism, like in the bellows figure:

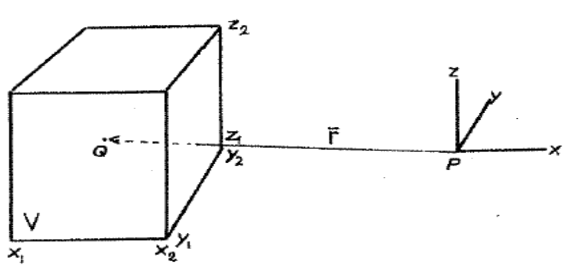

Fig. 4: Cartesian coordinate system {x,y,z} used for the evaluation of terrain correction of a rectangular prism

The computing formula used for terrain correction of a rectangular prism over the P point is the following (MacMillan 1958):

$$2TC = G\rho\left|\left|\left|2xy\log(z+r) + 2xz\log(y+r) + 2zy\log(x+r) - x^2 arctg\frac{yz}{xr} - y^2 arctg\frac{xz}{yr} - z^2 arctg\frac{xy}{zr}\right|_{x_2}^{x_1}\right|_{y_2}^{y_1}\right|_{z_2}^{z_1} \qquad (2)$$

Applying computed terrain corrections were determined the improved Bouguer anomalies in the gravimetric points.

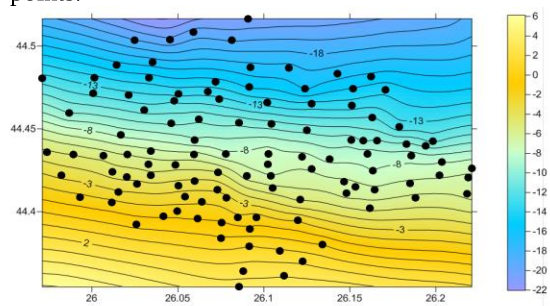

Fig. 5: AImproved Bouguer anomalies in the gravimetric points

d. Generation of improved Bouguer anomalies grid using the collocation / minimum curvature method

For this purpose was used a tension factor T=0.25 recommended by Smith and Wessel (1990).

e. Determination of long wavelength components in grid nodes using global geopotential model coefficients EGM2008

These elements were computed using the grids and software given by team which realized the EGM2008 model.

f. Calculation of residual gravity anomalies in the grid nodes

These were computed using relation:

$$\Delta g = \Delta gF - \Delta gEGM2008 \qquad (3)$$

g. Calculation of medium wavelength component of the geoid undulation in the grid nodes (NΔg)

Calculation of medium wavelength component of the geoid undulation in the grid nodes was realized using the Stokes formula.

h. Calculation of the indirect effect in the grid nodes (Nind)

This step was realized for close area and for remote area situated at a maximum distance of 50 km.

i. Calculation of the final values of geoid undulation in the grid nodes

The final values of the geoid undulations were obtained by correction of the mean wave-length component of the indirect effect and the addition of the long wavelength component, using the formula:

$$N = NEGM2008 + N\Delta g + Nind \qquad (4)$$

j.Calculation of the height anomalies in the grid nodes

For calculation of the height anomalies was used the relation developed in 1993 by Heiskanen and Moritz in ”Physical Geodesy”:

$$N - \zeta = \frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} H = H^* - H \quad (5)$$

According to the considerations made by the same authors, all previously cited in the paper, on page 327, it can be writed that:

$$\frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} = \frac{\Delta g_B}{981 gal} = 10^{-3} \Delta g_B \quad (6)$$

where Δg_B represents the Bouguer anomaly, in gal, and H represents the normal height, in meters.

The height anomalies are represented as follows (Fig. 6):

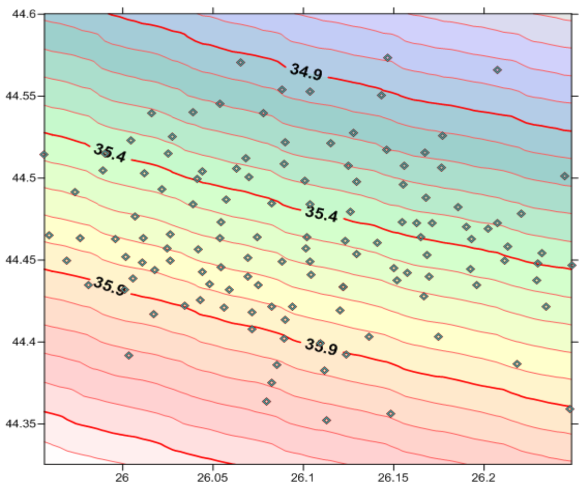

Fig. 6: Representation by isolines of height anomalies, corresponding to Bucharest area (Sorta, V – Doctoral thesis)

Based on height anomalies wre computed the following statistical parameters:

Table 1: Statistics of the height anomalies in the grid nodes

Statistics	Height anomalies [m]
Min.=	34.570
Max.=	36.672
Average=	35.618
Std. dev.=	0.503

3. Determination of the geometric quasigeoid for Bucharest area

The quasigeoid model for Bucharest area was determined through geometric method based on static GNSS determinations and levelling measurements realized in 121 gravimetric points. GNSS determinations were realized in one hour sessions, with 10 seconds registration rate.

For generating the geometric quasigeoid were used the relation:

ζ≈helipsoidal-Hnormal (7)

where ζ represents the height anomaly.

Based on height anomalies in measured points were computed the following statistics:

Table 2: Statistics of height anomalies in measured points

Statistics	Height anomalies [m]
Minim=	34.899
Maxim=	36.282
Media=	35.660
Ab. Standard=	0.281

4. Comparison between the geometric method and the gravimetric method for modeling the quasigeoid for Bucharest

Until now only one study was conducted in our country, in the Bucharest area, which can be carried out a comparison test between the geometric and the gravimetric method: Sorta, V. - Doctoral thesis - *”Contributions for a precise geoid/quasigeoid model for the area of Romania” – 2013.*

Considering the grid corresponding to the gravimetric quasigeoid, the height anomalies in the gravimetric points were interpolated.

On the other side, in the same points, were calculated the geometrical height anomalies, obtained as differences between ellipsoidal and normal heights.

After that, were determined the differences between gravimetric and geometric anomalies, given in the next table:

Table 3: Examples of values for height anomalies in common points computed using gravimetric method and comparison with the geometrical values

No.	absolute g [mgal]	γ [mgal]	$\zeta_{gravimetric}$ [m]	$\zeta_{geometric}$ [m]	Δζ [m]
1	980528,505	980577,028	35,174	35,157	0,017
2	980536,269	980569,776	35,775	35,777	-0,002
3	980537,132	980568,359	35,896	35,851	0,045
4	980539,302	980568,105	35,684	35,673	0,011
5	980542,180	980563,769	36,012	35,997	0,015
6	980529,111	980578,602	34,993	34,899	0,094

Taking into account these differences, were obtained the next statistics:

Table 4: Statistics of differences between geometric and gravimetric height anomalies

Statistics	Anomalies differences [m]
Min.=	-0.118
Max.=	0.127
Media=	0.002
Ab. Standard=	0.039

Considering the relation no. (7) and supposing that helipsoidal and Hnormal are independent variables, we can conclude the following relations for computing the geometrical height anomaly:

$$\sigma^2_{\zeta GPS/NIV} = \sigma^2_h + \sigma^2_H \qquad (8)$$

And for the height anomalies differences

$$\sigma^2_{\Delta\zeta} = \sigma^2_{\zeta Gravimetric} + \sigma^2_{\zeta GPS/NIV} \qquad (9)$$

This implies that the precision of gravimetric heght anomaly will be

$$\sigma^2_{\zeta Gravimetric} = \sigma^2_{\Delta\zeta} - \sigma^2_h - \sigma^2_H \qquad (10).$$

Considering the previous formulas we can conclude that the precision of the gravimetric quasigeoid model is about ±2,5 - 3,3 cm and the precision of the geometric quasigeoid model in GNSS determined points and precise levelling is about ±3,4-4,2 cm.

5. Conclusions

In most developed countries tend to achieve precise geoid models or quasigeoid, which served as a reference surface for a series of measurements and phenomenon, providing the main support for multiple engineering disciplines related to the Geosciences (navigation, seismology, tectonics study, the internal structure of the Earth, mapping, surveying, engineering surveying, etc.).
On the other hand, in Romania, measurements using GNSS technologies have experienced exponential growth in recent years. In this context, the National Agency for Cadastre and Land, through the Department of Geodesy and Mapping has developed a national system for determining the position, ROMPOS (Romanian position determination system), based on the latest generation of GNSS technology, being in agreement with European standards. Through this system, users can determine their 2D position in real time with high precision (of the order of centimeters). However, it still faces difficulties in determining altitudes (through GNSS measurements are obtained ellipsoidal heights which are not referred to a physical surface) and there is not a geoid model or quasigeoid viable for the national territory. Regarding the criteria of efficiency, ie the economic and precision, it is known that, in the conduct of gravimetric measurements, expenses, both material and human resources and time are reduced significantly compared with those allocated for a GNSS - leveling campaign. For example, to achieve measurements in Bucharest (relating to the present case study), while for gravimetric determinations was necessary a single operator (specializing in gravimetry) with one tool, GNSS - leveling measurements implied use of several devices together with mixed teams of specialists. In addition, the rough areas (hill and mountain), by lowering the leveling lines and the need to use trigonometric leveling measurements, observations accuracy drops significantly, being often cumbersome and also access to points where it wants to make measurements is difficult. Also, when performing GNSS observations may occur number of drawbacks, the main disadvantage consisting of lower accuracy for determining ellipsoidal heights.
Given the above, we can also conclude that establishing and maintaining precision leveling networks presents a great importance throughout Romania, which implies a coherent strategy in this regard of the relevant institutions, in particular National Agency for Cadaster and Land Registry. This network will have utility for gravimetric quasigeoid determination at national level and to perform other specialized works, such as works of photogrammetry, cartography, digital terrain models determinations, surveying and other technical works that require a precise determination of the altitudes. In this context, it is worth noting that currently there are no other methods to replace leveling where the values of altitudes are desired with millimeter precision. However, with the completion of gravimetric quasigeoid nationally, some functions will be taken over by leveling GNSS technology.

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