Hypothesis verification models in leveling geodetic networks

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Received: April 2015 / Accepted: June 2015 / Published: September 2015

Abstract

Verifications by calculating the amount "Stress values" in the characteristic sections of the building are confronted with the results of tests on models or test sections. These are - currently - followed by observations made by means of measures installed both in body building and outside it.

Hypothesis verification models are supplied by statistical tests. At measurements compensation are formulated several hypotheses. To validate or not the results obtained after compensation is necessary to verify such assumptions.

To detect possible deformations (displacement) occurring between two networks, observed at different times, it should be performed analysis only in the remaining common (identical) points in the interval T_i and T_{i+1}. In principle, comparing the coordinates (ie altitudes / heights) of the network points determined at different stages and investigates whether they form or not congruent figures. The difference between the determined parameters for network points should be within into a "safety margin".

The safety margin is calculated according to the empirical standard deviation. If this does not fall within safe limits, the statistic test does not indicate anything other than that in the network have appeared deformations.

Keywords

Geometric leveling, accurate measurements, compensation, stability, displacements, statistical tests.

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1. Introduction

Verifications by calculating the amount "Stress values" in the characteristic sections of the building are confronted with the results of tests on models or test sections. These are - currently - followed by observations made by means of measures installed both in body building and outside it. By comparing the results of these measurements performed on the construction with the results obtained by test of models (samples) and data obtained by calculations concerning the load capacity of building could be accomplished a "diagnosis" on the state of the building and eventually an estimate of its future behavior. This will allow any measures to strengthen or further use of such constructions.

The statistical hypotheses verification models are constituted by **statistical tests**. At compensation of measurements are formulated several hypotheses. In order to validate or not the results of compensation is necessary to verify the hypothesis in question. The main measures to be taken to verify them are:

- Verification of assumptions made on the parameters;
- Verification that the distribution corresponds to the respective distribution;

Statistical hypothesis is an assumption that can be **T** - true or F - false, depending on the assumed risk coefficient (α).

Statistical hypotheses verification algorithms are:

1. Ordering increasing these values;
2. The calculation of statistics (Θ) is made according to the nature of the size or depending on distribution:

$\theta = (t, \chi^2, F, Z)$

3. Defining the hypothesis H_0 the definitive hypothesis H_A:

H_0 -null hypothesis..

În cazul acestei ipoteze se verifică egalitatea a două mărimi $\theta_1 = \theta_2$:

H_A - alternative hypothesis;

In the case of this hypothesis we have the following situations: **unilateral** (single), **alternative** and **bilateral alternative**.

4. Establishing of the risk coefficient (generally in geodesy, the risk coefficient is 5%);

5. Determination of the critical statistics ($\theta_{critic} = \theta_{f,\alpha}$) which it extracted from the corresponding statistic's distribution tables depending on the coefficient of risk and degrees of freedom;

6. Comparison of the calculated value $\theta_{calculat}$ with the critical value $\theta_{critical}$. [1]

2. Statistical models

These are the simplest and most commonly models used in practice, in their analysis is interesting for us the status of the object at the time of measurement. At first, these models allow the analysis of only two stages of measurements with time they have improved. Currently, they allow a global analysis of several stages of measurements.

2.1 The global test of congruence

To detect possible deformations (displacements) occurring between two geodetic networks will be analyzed only remaining common points (identical points) between the interval T_1 and T_2.

In principle, is compared the coordinates of the network points determined at different stages and is examined whether they form or not congruent figures.

The difference between the determined parameters for network points must be within safety limit. The safety limit is calculated according to the **empirical standard deviation**. If the difference does not fall within safe limits, the statistic test does not indicate anything other than that, in the network have appeared deformations.

Through the global congruence test intended to determine a critical value, which is determined based on statistical distribution of measurements. This value we compare to the theoretical value of the statistical distribution under the same hypothesis that the critical value was calculated.

The inconformity with the safety limits of the parameters determined by processing, indicate that the two networks are not congruent.

The conditions that the test of congruence to locate the deformations in the network are:

- for both stages of observations should be entered the same provisional coordinates, being able to make reference to the same sizes, ie the same *datum*;
- in both stages should have the same defect for the reference dates (the same *datum*);
- usually, the used processing model is that of a free networks:
  - o in the case of unconstrained networks must be known the coordinates of two points;
  - o in the case of constrained networks must be known the coordinates of at least three points;
- the network configuration in the both stages should be the same;
- the theoretical standard deviation should be the same for both stages of measurements.

At each stage are made the measurements and on this base can be established a functional - stochastic model:

➢ The functional model (FM):

$$L_i + v_i = A_i * \hat{X}_i \quad (1)$$

➢ The stochastic model (SM):

$$\sum L_i = \sigma_0^2 * Q_{L_i} \quad (2)$$

The principle of the method of the smallest squares (LSM):

$$V^T PV = min \quad (3)$$

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A_1 & \emptyset \\ \emptyset & A_2 \end{bmatrix} * \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \end{bmatrix} \quad (4)$$

Note: *The configuration matrix (A) indicates that the measurements are uncorrelated.*

$$P = \begin{bmatrix} P_1 & \emptyset \\ \emptyset & P_2 \end{bmatrix} \quad (5)$$

From equation (2) gives:

$$\sum L = \sigma_0^2 * \begin{bmatrix} Q_1 & \emptyset \\ \emptyset & Q_2 \end{bmatrix} \quad (6)$$

Note: *"0" on the secondary diagonal of the matrix of the relations (4), (5), (6) indicates that the measurements are considered as uncorrelated between the two measured stages* [2].

2.2 Comparison of theoretical standard deviation in two stages

The comparing the theoretical standard deviation in the two phases is done through a statistical hypothesis, which requires the comparison of the empirical deviation of the two stages of measurements:

$$H_0: E\left\{\left(\frac{s_0^2}{T_1}\right)\right\} = E\left\{\left(\frac{s_0^2}{T_2}\right)\right\} \cong \sigma_0^2 \quad (7)$$

The analysis is done through the **Fischer statistical test**:

$$F_{theoretic} = \frac{(s_0^2)_{T_1}}{(s_0^2)_{T_2}} \quad (8)$$

Note: *has Fischer distribution because it is a ratio of square shapes.*

$$F_{theoretic} = F_{t_1, t_2, 1-\alpha} \quad (9)$$

$t_i -$ no. of degrees of freedom;

$(1-\alpha) -$ safety threshold used by geodesists;

The test decision $F_{thoretic} \square F_{critical}$:

- If: $F_{theoretic} \geq F_{critical} \rightarrow$

$$H_0: \left(\frac{s_0^2}{T_1}\right) = \left(\frac{s_0^2}{T_2}\right) \cong \sigma_0^2 \quad (10)$$

- If: $F_{theoretic} < F_{critical} \rightarrow H_0 - false \rightarrow H_1 - true$

$$H_1: \left(\frac{s_0^2}{T_1}\right) \neq \left(\frac{s_0^2}{T_2}\right) \neq \sigma_0^2 \quad (11)$$

Be the normal system of equations (i=1, 2):

$$\overbrace{A^T{}_i P_i A_i}^{N} \widehat{X_1} = \overbrace{A^T{}_i P_i L_i}^{n} \quad (12)$$

$$\widehat{X}_i = -N^{-1}{}_i n_i \quad (13)$$

Note: *Since this is a free network there are no fixed points, so the configuration matrix has a rank defect which is transmitted to the matrix N. Therefore, the normal matrix determinant is zero, which means that it cannot be reversed. It follows that equation (13) cannot be applied:*

$det N = 0 \rightarrow \nexists N^{-1} \rightarrow N^{+}$ - pseudo reverse (14)

$$\widehat{X}_i = N^{+}{}_i n_i \quad (15)$$

$$N^{+}{}_i = (Q_x)_i \quad (16)$$

Solving the system (15) leads to finding the unknown parameters and corrections, with which we can calculate accuracies:

$$\frac{V^T{}_i P_i V_i}{n-h+d} = (s_0^2)_i \quad (17)$$

We make the notations:

$$V^T{}_i P_i V_i = \Omega_i \quad (18)$$

$$\Omega = \Omega_1 + \Omega_2 \quad (19)$$

$$n_i - h_i + d_i = f_i \quad (20)$$

$f -$ degrees of freedom (21)

$$s_0^2 = (s_0^2)_1 + (s_0^2)_2 \quad (22)$$

From relations (17), (18) and (20):

$$(s_0^2)_i = \frac{\Omega_i}{f_i} \quad (23)$$

From relations (19), (21), (22) and (23):

$s_0^2 = \frac{\Omega}{f}$ - deformation model (24)

2.3 Application of the statistical test when the configurations are identical [4]

To apply any statistical test should be fixed - previously - a hypothesis:

$$BX = W \quad (25)$$

B - the matrix that explains the function, also called configuration matrix;

W - the vector of discrepancies.

In the case of congruence test the hypotheses are made:

$$E\{\widehat{X_1}\} = E\{\widehat{X_2}\} \quad (26)$$

$$E\{\widehat{X_1}\} - E\{\widehat{X_2}\} = 0 \quad (27)$$

Through the hypotheses introduced by (26) and (27) we want to know how to change the value of Ω when the deformations appear.

$$\underbrace{[-1 \quad 1]}_{B} * \underbrace{\begin{bmatrix} \widehat{X_1} \\ \widehat{X_2} \end{bmatrix}}_{X} = \underbrace{0}_{W} \quad (28)$$

$$\Omega + R = \Omega_H \quad (29)$$

Ω_H - the value of Ω affected by hypothesis

Ω - initial value influenced by statistical hypothesis;

R - the amount that they contribute the statistical hypothesis introduced by us;

From equation (29), the amount of R will be determined as an error function, known from mathematical statistics:

$$F = f^T Q_f F \quad (30)$$

$$R = (BX - W)^T [B(A^T PA)^{+} B^T]^{+} (BX - W) \quad (31)$$

From equations (27) and (31) results:

$$R = (\widehat{X_2} - \widehat{X_1})^T [(A_1^T P_1 A_1)^{+} + (A_2^T P_2 A_2)^{+}]^{+} (\widehat{X_2} - \widehat{X_1}) \quad (32)$$

We make the following notations:

$$(\widehat{X_2} - \widehat{X_1}) = d$$
$$(A_1^T P_1 A_1) = Q_1 \quad (33)$$
$$(A_2^T P_2 A_2) = Q_2$$
$$Q_{dd} = Q_1^{+} + Q_2^{+}$$

We introduce the notations in (32) and obtain:

$$R = d^T (Q_{dd})^{+} d \quad (34)$$

where do you:

Q_{dd} - cofactors matrix of deformation model;

d - the discrepancy vector;

rang Q_{dd} = rang Q_1 = rang Q_2 = h;

h = n - d, n - number of measurements and d - rank defect.

The global test of congruence relation has the following form (he has Fischer distribution because it is a ratio of square shapes):

$$F = \frac{R}{s_0^2 h} \quad (35)$$

From the relations (34) and (35) we have:

$$F = \frac{d^T (Q_{dd})^{+} d}{s_0^2 h} \quad (36)$$

We introduce the equation (24) in the above relation and obtain:

$$F = \frac{d^T (Q_{dd})^+ d}{\frac{\Omega}{f} h} = \frac{d^T (Q_{dd})^+ d}{V^T P V} \frac{f}{h}$$
(37)

ie the:

$$F = \frac{R}{\Omega} \frac{f}{h}$$
(38)

Decision of the test:

$$F_{critical} = F = \frac{R}{s_0^2 h}$$
$$P\{F \geq F_{h,f,\alpha}\} = \alpha$$
$$F_{theoretic} = F_{h,f,\alpha}$$

- If: $F_{theoretic} \geq F_{critical} \rightarrow H_0 - true$

$$H_0: E\{\widehat{X_1}\} = E\{\widehat{X_2}\} \rightarrow \widehat{X_1} = \widehat{X_2} \rightarrow \nexists deformations$$
(39)

- If: $F_{theoretic} < F_{critical} \rightarrow H_0 - false \rightarrow H_1 - true$

$$H_1: E\{\widehat{X_1}\} \neq E\{\widehat{X_2}\} \rightarrow \widehat{X_1} \neq \widehat{X_2} \rightarrow \exists deformations$$
(40)

Remarks:

1. The decision by this test statistic is true with probability P = 1-α, ie P = 95%. It is not possible to take a decision that is certain, ie the P = 100%;

2. The congruence test reveals that the two networks are congruent or not. If not congruent, this test does not specify points "moved" that contributed to non-congruent networks.

2.4 Application of statistical test when the configurations are different [3]

To solve the problem, we will minimize the "*follow*" of the configuration matrix only for common points and then we will follow the above steps. Thus, the matrix G will be replaced by a *selection matrix* B, which relates only to the common points:

$$B = EG$$
(41)

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{bmatrix} \qquad (42)$$

Note: *The matrix E is formed by filling the main diagonal with digits 1 and 0 as follows: 1 – in the right of common points and 0 - in the right points which are not common.*

The matrix G is used to border on the right and bottom of the sub-matrices $A_i^T P_i A_i$, to calculate the pseudo-reverse N^+:

$$A^T{}_i P_i A_i = N_i \qquad (43)$$

$$\begin{bmatrix} N & \vdots & G \\ \dots & \vdots & \dots \\ G^T & \vdots & 0 \end{bmatrix}^{-1} = \begin{bmatrix} N^+ & \vdots & G(G^T G)^{-1} \\ \dots & \vdots & \dots \\ (G^T G)^{-1} G^T & \vdots & 0 \end{bmatrix}^{-1} \qquad (44)$$

Equation (43) is used when the configurations are the same. In case the configurations are different, the matrix B will replace the matrix G and the relation (43) becomes:

$$\begin{bmatrix} N & \vdots & B \\ \dots & \vdots & \dots \\ B^T & \vdots & 0 \end{bmatrix}^{-1} = \begin{bmatrix} N^+ & \vdots & G(BG)^{-1} \\ \dots & \vdots & \dots \\ (GB)^{-1} G^T & \vdots & 0 \end{bmatrix}^{-1} \qquad (45)$$

The matrix G must satisfy the following condition:

$$NG = 0 \qquad (46)$$

From the spectral analysis is demonstrated that the matrix G contains the eigenvectors of the configuration matrix, which has its own value 0. The number of eigenvectors corresponding to the rank defect of the monitoring geodetic network:

$$G^T X = 0 \qquad (47)$$

Note: *The form of the G matrices depends on the type of geodetic network: leveling, trilateration, two-dimensional or three-dimensional triangulation.*

Example:

❖ For leveling networks that have one degree of freedom (one translational direction H) the rank defect is d = 1 and the G matrix form is:

$$G = [1 \quad 1 \quad \cdots \quad \cdots \quad 1] \qquad (48)$$

=> no. of the vector elements = no. of the points in the network.

❖ For bi-dimensional triangulation networks, which have 7 degrees of freedom (three translations X, Y, Z, 3 rotations around the Y and Z axes and the scale factor **m**) the rank defect is d = 7 and the G matrix form is:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & H_A^0 & -Y_A^0 & X_A^0 \\ 0 & 1 & 0 & -H_A^0 & 0 & X_A^0 & Y_A^0 \\ 0 & 0 & 1 & Y_A^0 & -X_A^0 & 0 & H_A^0 \\ 1 & 0 & 0 & 0 & H_B^0 & -Y_B^0 & X_B^0 \\ 0 & 1 & 0 & -H_B^0 & 0 & X_B^0 & Y_B^0 \\ 0 & 0 & 1 & Y_B^0 & -X_B^0 & 0 & H_B^0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \qquad (49)$$

❖ For the three-dimensional triangulation networks, which have 4 degrees of freedom (2 translations X, Y, 1 rotation around the H axis and the scale factor **m**) the rank defect is d = 4 and the G matrix form is:

$$G = \begin{bmatrix} 1 & 0 & Y_A^0 & X_A^0 \\ 0 & 1 & -X_A^0 & Y_A^0 \\ 1 & 0 & Y_B^0 & X_B^0 \\ 0 & 1 & -X_B^0 & Y_B^0 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

(50)

❖ For the trilateration networks and heterogeneous networks, which have 3 degrees of freedom (2 translations X, Y, 1 rotation around the H axis) the rank defect is d = 3 and the G matrix form is:

$$G = \begin{bmatrix} 1 & 0 & Y_A^0 \\ 0 & 1 & -X_A^0 \\ 1 & 0 & Y_B^0 \\ 0 & 1 & -X_B^0 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{bmatrix}$$

(51)

2.4 Localizarea deformațiilor utilizând testul Student „t”

$$t_{critic} = t_j = \frac{d_j}{s_j}$$

(52)

$$s_j = s_0 \sqrt{Q_{dd_i}}$$

(53)

$$t_{limita} = t_{f,1-\alpha}$$

(54)

The test decision:

$$P\{t \geq F_{f,1-\alpha}\} = \alpha$$

$$t_{teoretic} = t_{f,1-\alpha}$$

If: $t_{theoretic} \geq t_{critic} \rightarrow j - fixed$

If: $t_{theoretic} < t_{critic} \rightarrow j - displaced$

3. Case Study

To demonstrate the theoretical notions presented above we used the measurements performed in a geodetic leveling network, network of monitoring in time of an important engineering construction, namely the People's Salvation Cathedral.

In this network were carried out successive measurements of geometric precision leveling in order to determine the vectors of movement for the floating marks (leveling, settling marks) mounted on the structural elements of the building in construction.

For the case study were considered a number of 3 stages of measurements made on the 4 **reference marks**, located outside the zone of influence of the construction works.

3.1 Processing the measurements

To determine with high accuracy the heights of the points materialized on the building which is subject to monitoring (***settling marks***) was considered as the only solution to the problem is to design a network of reference, consisting of at least **4 landmarks**, located outside the zone of influence of the construction execution in which there is the possibility of performing accurate measurements to the settling marks.

In this article, these points are called **RA1**, **RA2**, **RA3** and **RA4** (Figure 1).

The determination of the accurate level differences between stations points, was ensured with the use of geometric precision leveling instrument (the digital leveling instrument) ***TOPCON DL-101C***.

For readings were used encoded leveling rods with invar band. In these conditions, the system ensures an accuracy of 0.3 - 0.4 mm / double km of leveling.

Were measured five level differences, resulting in a system of 5 equations of corrections.

The processing (compensation) of measurements was performed by use the concept of *free network* (without constraints).

The compensation calculations and determination of heights (H) of points were performed and the global test of congruence and localization of deformation was applied (if applicable) for each of the 3 stages of measurements (Table 1).

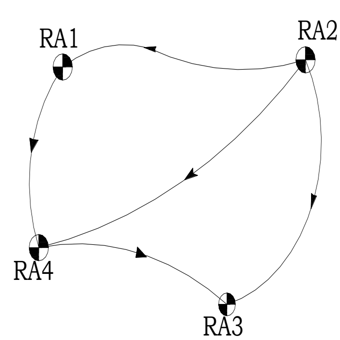


Fig. 1 The scheme of precision leveling network

Table 1. The heights of the landmarks in the 3 stages of measurement

Mas. stage	STAGE 0 29.06.2012	STAGE 1 10.09.2012		STAGE 2 10.01.2014		
Pts. No.	H (m)	H (m)	[illegible]	Pts. No.	H (m)	H (m)
RA1	85,5502	85,5501	-0,1	85,5434	-6,8	-6,7
RA2	88,7188	88,7189	0,1	88,7166	-2,2	-2,3
RA3	87,6853	87,6854	0,1	87,6904	5,1	5
RA4	86,7895	86,7893	-0,2	86,7933	3,8	4

Following these tests has been reached the following conclusions highlighted in the monitoring sheet.

3.2 The results of statistical tests

The detection of deformations (**Fisher** test):

	s_0 [mm]	f	h
etapa 1	0,63	4	3
etapa 2	1,62	4	3
etapa 3	41,71	4	3

	f	h	s0 [mm]	F computed	F critical	Notes
Stage 1-2	4	3	1,7	34,2822	6,5914	deformations
Stage 1-3			41,7	82,4028		deformations
Stage 2-3			41,7	82,2974		deformations

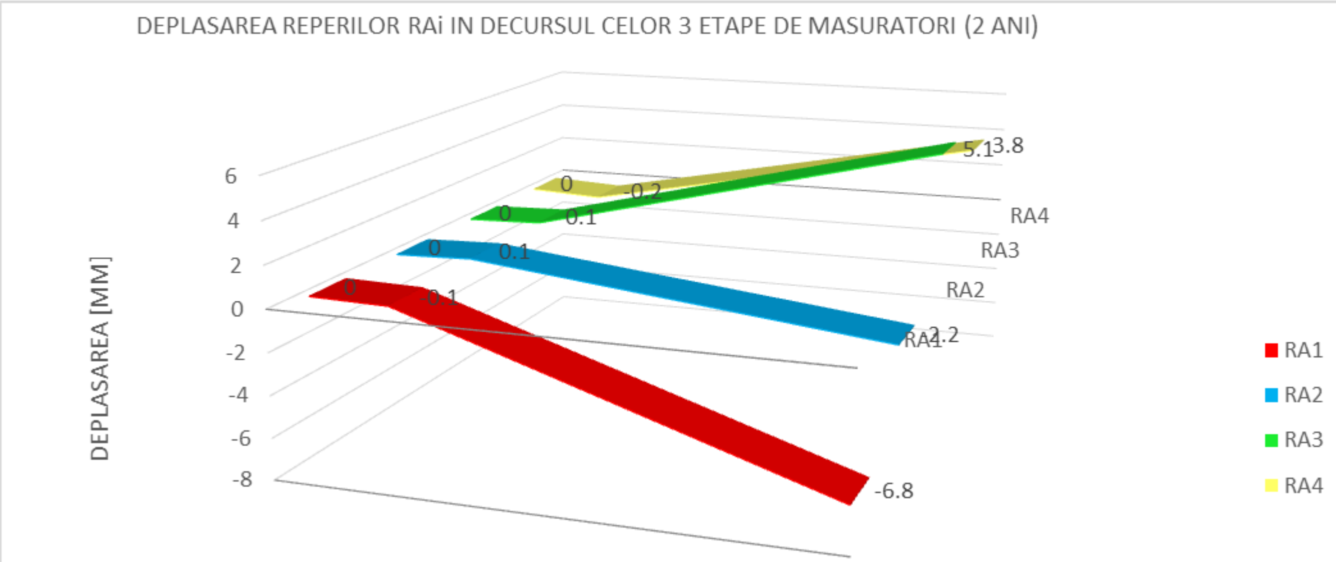
Locating of deformations (**Student** "**t**" test):

Stage 1-2				
Pts. No.	**sj**	**tj**	**t**	**Notes**
1	0,7316	-15,7714		displaced
2	0,6542	4,8712	2,1318	displaced
3	0,8311	8,6650		displaced
4	0,6046	-3,2890		displaced

Stage 2-3				
Pts. No.	**sj**	**tj**	**t**	**Notes**
1	17,58552	-2,62992		displaced
2	15,72473	-6,67859	2,1318	displaced
3	19,97837	3,970532		displaced
4	14,53323	3,624652		displaced

Stage 1-3				
Pts. No.	**sj**	**tj**	**t**	**Notes**
1	17,57427	-2,61013		displaced
2	15,71467	-7,07803	2,1318	displaced
3	19,96559	3,893766		displaced
4	14,52393	3,796387		displaced

The results of applying the statistical tests, namely the vertical displacement vectors of the reference and control points can be seen in the suggestively chart below.



	26/06/2012	10/09/2012	10/01/2014
RA1	0	-0.1	-6.8
RA2	0	0.1	-2.2
RA3	0	0.1	5.1
RA4	0	-0.2	3.8

Conclusions

In the case of in time monitoring of constructions the purpose is to determine if the building or the materialized points of respective building were moved or not. If they were displaced the following questions appearing:

1. This displacement represents or not a danger for the respective construction works?
2. In what place was performed the displacement?

Thus, in order to answer these questions, with a probability of 95% applies statistical tests described above. After applying these tests we can find the answer to the questions above and we will be able to prevent potential material damages or even save human lives.

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